ST 305 HW 7 Solution
(Total points: 44)

Ch 6.2: 40, 42, 46, 50, 51, 52, 53, 54, 55, 66, 68, 70, 77, 78

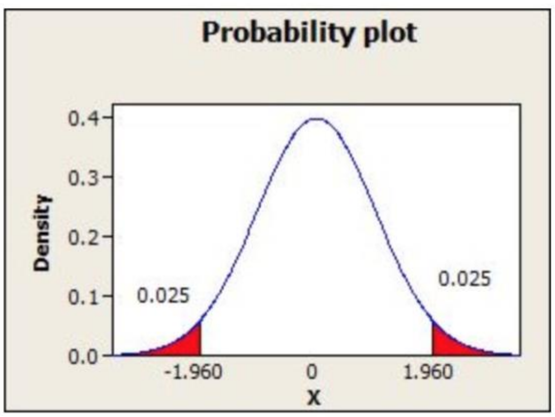
40. (2 points, one for each)
(a)

Find the value of *z* in a two-sided significance test where $P = 0.05$

Since it is a two sided test, the total area under both tails is 0.05, and so each tail has only half that area, 0.025.

Using Table A, find the nearest approximation to 0.025 in the area portion.

The graph is as shown below:



Since the Normal curve is symmetric, $P(z \le -1.96) = P(z \ge 1.96)$ and so the right value of *z* is $z = 1.96$.
(b)

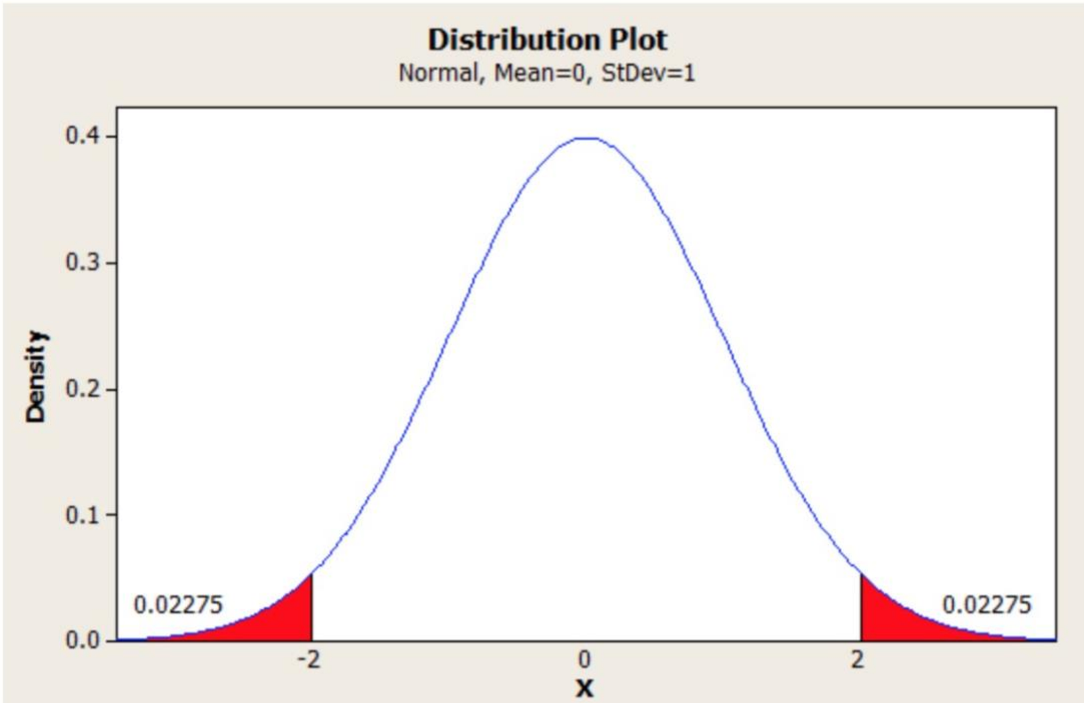
The aim is to identify the significant *z* value from the above graph.

The values that are statistically significant at the $\alpha = 0.05$ level are the values where $P(z \le -1.96)$ and $P(z \ge 1.96)$

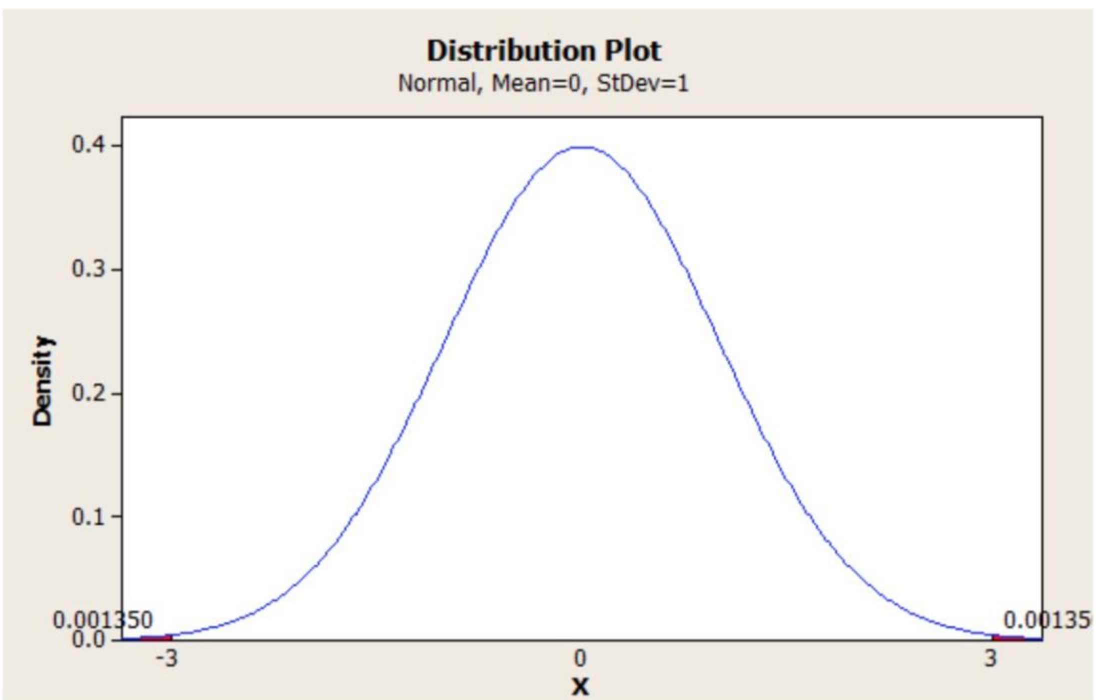
Hence, the value above 1.96 and below -1.96 are significant.

42. (2 points, one for each)

The graphical representation for the *z*-score = 2 of two-tailed test is as follows,



The graphical representation for the *z*-score = 3 of two-tailed test is as follows,



Here, observe that for two-tailed test, the two *z*-scores 2 and 3 are lies within 3σ limits. Therefore, we can conclude that the Supreme Court can claim that the *z*-scores $z^* = 2$ and $z^* = 3$ are generally convincing statistical evidence at **99% significance level**.

46. (2 points, one for each)
It is given that there is a 95% confidence that the true mean lies between 53 and 62.
(a) $H_0: \mu = 58$ vs $H_a: \mu \neq 58$
Since $\mu_0 = 58$ lies within the confidence interval (53,62), we cannot reject H_0.
(b) $H_0: \mu = 63$ vs $H_a: \mu \neq 63$
Since $\mu_0 = 63$ lies outside the confidence interval (53,62), we can reject H_0.

50. (4 points, one for each)
A. When testing a null hypothesis, μ_0 is used. $\bar{x}$ is the mean of the observed values.
B. Given $n = 30, \sigma = 5$ the standard deviation of the sample mean would use the formula, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. Therefore, the standard deviation of the sample mean would be $\sigma_{\bar{x}} = \frac{5}{\sqrt{30}}$ which is approximately 0.9129
C. The study needs to use $\mu = \mu_0$ when reporting statistical significance, not the sample mean $\bar{x}$.
D. This should read, A researcher test they hypothesis $H_0: \mu = 350$ and concludes that the sample mean is equal to 350.

51. (4 points, one for each)
(a) A significance test does not reject a null hypothesis based on the sample mean. It must be based on the population mean.
(b) If the company wants to test that the average score on the ACT is better than the national average. The null hypothesis should be that there is no change and that alternative would be $H_a: \mu > 21.2$.
(c) The p-value for this study is much too large to be statistically significant. The smaller the p-value the more significant it is.
(d) When determining whether to reject a hypothesis or not, compare the p-value to α never the z value.

52. (3 points, one for each)
A. The null hypothesis should be:

H_0: The percentage of student who own cell phones is 88%, $\mu = 88$

And the alternative hypothesis should be:

H_a: The percentage of students who own a cell phone is greater than 88%, $\mu > 88$
B. The null hypothesis should be:

H_0: The students in the morning recitation section have the same mean score as the whole class, $\mu = 75$

And the alternative hypothesis should be:

H_a: The students in the morning recitation section have a higher mean score than the whole class, $\mu > 75$

C. The null hypothesis should be:

H_0 : The paper is the same as the old, $\mu = 0$

And the alternative hypothesis should be:

H_a : The paper is different than the old, $\mu \neq 0$

53. (3 points, one for each)

A. The null hypothesis should be:

H_0 : The average score is a 77%, $\mu = 77$

And the alternative hypothesis should be:

H_a : The average score differs from 77%, $\mu \neq 77$

B. The null hypothesis should be:

H_0 : The average time to complete a maze is 20 second, $\mu = 20 \sec onds$

And the alternative hypothesis should be:

H_a : The time to complete a maze while rap music is playing will take more than 20 second, $\mu > 20 \sec onds$

C. The null hypothesis should be:

H_0 : The average square footage of a one bedroom apartment is advertised as 880 square feet, $\mu = 880 ft^2$

And the alternative hypothesis should be:

H_a : The apartments are actually less than advertised, $\mu < 880 ft^2$

54. (3 points, one for each)

(a) Null hypothesis: male MTV viewers is equal to female MTV viewers.
Alternative hypothesis: male MTV viewers is greater than female MTV viewers.

(b) Null hypothesis: Positive attitudes result in the same mean scores.
Alternative hypothesis: Positive attitudes result in higher mean scores than neutral attitudes.

(c) Null hypothesis: Time spent on social network sites has either no effect or positive effects on self-esteem.
Alternative hypothesis: Time spent on social network sites negatively effects self-esteem.

55. (2 points, one for each)

A. The null hypothesis should be:

H_0 : The mean income is $42,800 per household per year, $\mu = 42,800$

And the alternative hypothesis should be:

H_a : The mean income of mall shoppers is higher than the general population, $\mu > 42,800$

B. The null hypothesis should be:

H_0: Online registration took an average of 0.4 hours to respond to calls,

$\mu = 0.4 hours$

And the alternative hypothesis should be:

H_a: Response time differs from last years, $\mu \neq 0.4 hours$

66. (1 point)

The null and alternative hypotheses are,

The population mean of the distribution of longleaf pine trees in the wade tract is100.

$H_0 : \mu = 100$

$H_a : \mu \neq 100$

Here, μ is the hypothesized mean.

Let the sample mean be $\bar{x} = 99.74$.

Let the population standard deviation be $\sigma = 58$.

The test statistic is,

$$
\begin{aligned}
z &= \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \\
&= \frac{99.74 - 100}{\frac{58}{\sqrt{584}}} \\
&= -0.108331 \\
&\approx -0.11
\end{aligned}
$$

Find the P – value.

$$
\begin{aligned}
P &= 2P(Z \geq |z|) \\
&= 2P(Z \geq |-0.11|) \\
&= 2P(Z \geq 0.11) \\
&= 2[1 - P(Z < 0.11)] \\
&= 2[1 - 0.5438] \\
&= 2(0.4562) \\
&= 0.9124
\end{aligned}
$$

The *P*-value is greater than any level of significance. Fail to reject the null hypothesis. Therefore, it can be concluded that the mean value is equal to 100.

68. (3 points, one for z-test, one for p-value and one for conclusion)

The sample size $n = 6$.

The sample mean $\overline{x} = 10.2$

Population standard deviation $\sigma = 2.5$

The Population mean $\mu = 8.9$

The aim is to test whether there is evidence that the average new sonnets are greater than 8.9

Null hypothesis:

The average new words in sonnets is equal to 8.9

$H_0 : \mu = 8.9$

Alternative hypothesis:

The average new words in sonnets are greater than 8.9

$H_1 : \mu > 8.9$

Test-Statistic:

$$Z = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$
$$= \frac{10.2 - 8.9}{2.5 / \sqrt{6}}$$
$$= \boxed{1.27}$$

The value of *P*, from MS-Excel is $\boxed{0.1010}$

Since the value of *P* is greater than $\alpha = 0.05$, fail to reject Null hypothesis H_0.

Hence, conclude that the average new words in sonnets is equal to 8.9

70. (2 points, one for each)

a)

The null hypothesis should be:

H_0: The recommended amount of caloric intake, $\mu = 2,811.5 kcal / day$

And the alternative hypothesis should be:

H_a: The caloric intake is lower than the recommended amount, $\mu < 2,811.5 kcal / day$

b)

Given $\mu = 2811.5, n = 201, \sigma = 880, \overline{x} = 2403.7$ find the P-value by first finding the test statistic.

Test statistic,

$$z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}$$
$$= \frac{2,403.7 - 2,811.5}{880 / \sqrt{201}}$$
$$= -6.57$$

Find the P-value if $H_a : \mu < 2,811.5$ by using $P = P(Z \leq -6.57)$

Find $P = P(Z \leq -6.57)$ by using Standard Normal probabilities table. It was found that

$P(Z \leq -6.57) < 0.0003$ and so

$P(Z - 6.57) \approx 0$

$\boxed{P \approx 0}$

The data is significant at a level $\alpha = .001$ and therefore the null hypotheses can be rejected.

77. (1 points)

To be significant at a 5% level an event will happen fewer than 5 times in 100 whereas to be significant at a 10% level an event will happen fewer than 10 times in 100. If something happens fewer than 5 times it also happens less than 10 times and therefore anything significant at a 5% level must always be significant at the 10% level too.

78. (1 points)

To be significant at a 5% level an event will happen fewer than 5 times in 100 whereas to be significant at a 1% level an event will happen less than 1 time in 100. If something happens less than 1 time it also happens less than 5 times and therefore anything significant at a 1% level must always be significant at the 5% level too. However, it cannot be determined if something that is significant at the 5% level is also significant at the 1% level

Ch 6.3: 90, 93, 100

90. (1 points)

The objective is to test whether vitamin C has a strong effect in preventing colds or not.

From the given information,

The P – value is 0.03.

Let level of significance be $\alpha = 0.05$.

State the null and alternative hypotheses.

Null hypothesis: Vitamin C has no effect in preventing colds.

Alternative hypothesis: Vitamin C has strong effect in preventing colds.

Decision rule:

Reject the null hypothesis, if the $P-$value is less than the level of significance $\alpha = 0.05.$

Conclusion:

Since, $0.03 < 0.05,$ that means, the $P-$value is less than the level of significance $\alpha = 0.05.$ Since, reject the null hypothesis and it is conclude that Vitamin C has strong effect in preventing colds.

93. (3 points, one for each)

a)

Given $\mu = 2403.7, n = 100, \sigma = 880, \bar{x} = 2453.7$ find the P-value by first finding the test statistic.

Substitute in known values for the test statistic:

$$\begin{aligned} z &= \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \\ &= \frac{2,453.7 - 2,403.7}{880 / \sqrt{100}} \\ &= 0.57 \end{aligned}$$

Find the P-value if $H_a : \mu > 2,403.7$ by using $P = P(Z \geq 0.57)$

Find $P = P(Z \geq 0.57)$ by using Standard Normal probabilities table. It was found that $P(Z \leq 0.57) = 0.7157$ and so

$$\begin{aligned} P\text{-value} &= P(Z \geq 0.57) \\ &= 1 - 0.7157 \\ &= \boxed{0.2843} \end{aligned}$$

b)

Given $\mu = 2403.7, n = 500, \sigma = 880, \bar{x} = 2453.7$ find the P-value by first finding the test statistic.

Substitute in known values for the test statistic:

$$\begin{aligned} z &= \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \\ &= \frac{2,453.7 - 2,403.7}{880 / \sqrt{500}} \\ &= 1.27 \end{aligned}$$

Find the P-value if $H_a: \mu > 2,403.7$ by using $P = P(Z \geq 1.27)$

Find $P = P(Z \geq 1.27)$ by using Standard Normal probabilities table. It was found that $P(Z \leq 1.27) = 0.8980$ and so

$$\begin{aligned} P\text{-value} &= P(Z \geq 1.27) \\ &= 1 - 0.8980 \\ &= \boxed{0.102} \end{aligned}$$

c)

Given $\mu = 2403.7, n = 2500, \sigma = 880, \bar{x} = 2453.7$ find the P-value by first finding the test statistic.

Substitute in known values for the test statistic:

$$\begin{aligned} z &= \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \\ &= \frac{2,453.7 - 2,403.7}{880 / \sqrt{2500}} \\ &= 2.84 \end{aligned}$$

Find the P-value if $H_a: \mu > 2,403.7$ by using $P = P(Z \geq 2.84)$

Find $P = P(Z \geq 2.84)$ by using Standard Normal probabilities table. It was found that $P(Z \leq 2.84) = 0.9977$ and so

$$\begin{aligned} P\text{-value} &= P(Z \geq 2.84) \\ &= 1 - 0.9977 \\ &= \boxed{0.0023} \end{aligned}$$

100. (1 points)

From the information, you can perform 1000 significance tests using $\alpha = 0.05$.

Here, the Null hypotheses are true.

So, we can say that 1000×0.05=50.

We would expect about 50 of these tests to be labeled "statistically significant" out of 1000 attempts.

Ch 6.4: 104, 110, 111

104. (1 points)

If there is a power of 35%, then there is a 35% chance of rejecting the null when at alternative is true. This is not a strong power and therefore the study should not be run. One option to raise the power is to increase the sample size.

110. (4 points, one for each)

a)

Using standard normal probability tables, the z-score which is closest to 0.95 is $z = 1.645$ any *z*-value greater than or equal to 1.645 will be rejected at a 5% significance level. Therefore, any $\boxed{z \geq 1.645}$ is rejected

b)

Using $n = 70, \sigma = 27, \mu = 168, z \geq 1.645$ find which values of $\bar{x}$ will reject the null hypothesis.

Under the null hypothesis the test statistic is defined as,

$$z \geq \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

$$1.645 \geq \frac{\bar{x} - 168}{27 / \sqrt{70}}$$

$$1.645 \geq \frac{\bar{x} - 168}{3.23}$$

$$\boxed{\bar{x} \geq 173.31}$$

All values greater than or equal to 173.31 will reject the null hypothesis.

c)

We find the probability that $\bar{x} \geq 173.31$ will be rejected when the alternative $\mu = 173$

$$P(\bar{x} \geq 173.31 \text{when} \mu = 173) = P\left(\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \geq \frac{173.31 - 173}{27 / \sqrt{70}}\right)$$
$$= P(Z \geq 0.096)$$
$$= \boxed{0.5382}$$

Therefore, there is a nearly 54% chance of rejecting the null hypothesis when the true mean is 173.

d)

The higher the power is, the higher the percentage rate of rejection. This power is somewhat weak and they should find ways to increase it. A couple options for that sample size are, decrease the standard deviation, increase the significance level or consider an alternative further from 168.

111. (1 points)

Power At $\mu = 462$

$$= P(Z \geq 2.326 / \mu = 462)$$

$$= P\left(\frac{\overline{X} - 450}{100/\sqrt{500}} \geq 2.326 / \mu = 462\right)$$

$$= P(\overline{x} \geq 460.40 / \mu = 462)$$

$$= P\left(\frac{\overline{X} - 462}{100/\sqrt{500}} \geq \frac{460.40 - 462}{100/\sqrt{500}}\right)$$

$$= P(z \geq -0.36)$$

$$= P(z \leq 0.36) \qquad \text{[By symmetry of Normal distribution]}$$

$$= 0.6406$$

In order to declare the test is sensitive, the probability of rejecting H_o when $\mu = 462$ should be high; here it is considerably large so the test is sufficient to detect an increase of 12 points.