ST 305 Hw6 Solution
(Total: 19 points)

Ch 6.1: 10, 11, 12, 13, 14, 16, 17, 22, 23, 31,35

6.10 (2 points, one for each)

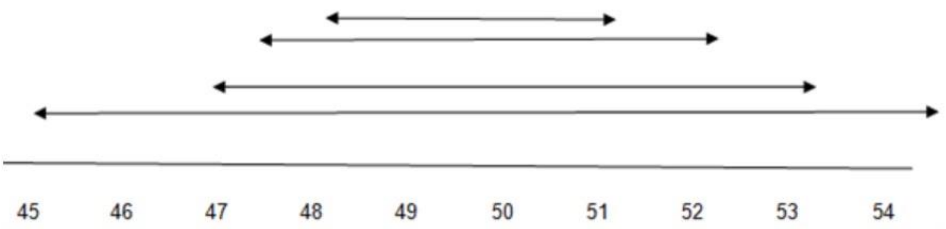
**6.10. (a)** The 95% confidence interval is $87 \pm 10 = 77$ to 97. (The sample size is not needed.)

**(b)** Greater than 10: A wider margin of error is needed in order to be more confident that the interval includes the true mean.

**Note:** *If this result is based on a Normal distribution, the margin of error for 99% confidence would be roughly 13.1, because we multiply by 2.576 rather than 1.96.*

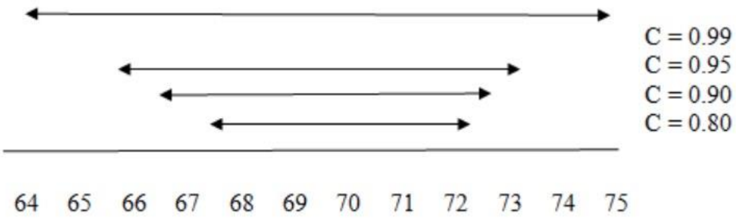
6.11 (1 point)

To illustrate the effect of sample size on the width of a confidence interval, a diagram like the one below may be helpful.




The top double-pointed arrow represents $n = 80$. The second arrow represents $n = 40$. The third arrow represents $n = 20$. The bottom arrow represents $n = 10$. As illustrated, as sample size increases, the width of the confidence interval decreases.

6.12 (1 point)




The top arrow represents $C = 0.99$. The second arrow represents $C = 0.95$. The third arrow represents $C = 0.90$. The bottom arrow represents $C = 0.80$. As illustrated, as confidence level increases, the width of the confidence interval increases.

6.13 (4 points, one for each)

a)

If the 95% confidence interval was calculated using the formula $\bar{x} \pm m$, then the formula $8.6 \pm 1.96(2.0)$ does not use a correctly calculated margin of error. The correct formula for margin of error is $m = z^* \frac{\sigma}{\sqrt{n}}$, so would be $(1.96)\frac{2.0}{\sqrt{400}} = 0.196$.

b)

This is an incorrect statement of the meaning of a confidence interval. We are not 95% confident that the sample mean falls within the confidence interval. Instead, we are confident that the *population* mean will fall within the confidence interval 95% of the time. The numbers are correct, but the confidence interval is used to estimate $\mu$, not $\bar{x}$.

c)

This is incorrectly states the meaning of a confidence interval. The confidence level is not a probability and should not be referred to as such. She can state that she is confident that the population mean will fall within the confidence interval 95% of the time.

d)

The sample size is large, which allows for an assumption, but not the one stated. When the sample size is large, we can be confident that the *sample mean* will be approximately , not the population.

6.14 (3 points, one for each)

**6.14.** **(a)** The standard deviation should be divided by $\sqrt{100} = 10$, not by 100. **(b)** The correct interpretation is that (with 95% confidence) the *average* time spent at the site is between 3.71 and 4.69 hours. That is, the confidence interval is a statement about the population mean, not about the individual members. **(c)** To halve the margin of error, the sample size needs to be quadrupled, to about 400. (In fact, $n = 385$ would be enough.)

6.16 (1 point)

The confidence interval for the mean score can be calculated on the assumption that the population is normally distributed.

The probability distribution for the integers 1 through 10 is approximately Normal. Even if this was not the case, the very large sample size of $n = 2673$ would make us confident that the sample distribution would be approximately Normal.

6.17 (2 points, one for margin of error and one for confidence interval)

Given that in a study of bone turn over in young women, serum "Trap" was measured in 31 subjects. The units are units per liter $(U/l)$. the mean was 13.2 $U/l$. The standard deviation is known to be 6.5 $U/l$.

Margin of error for 95 % confidence interval is $m = z^* \frac{\sigma}{\sqrt{n}}$

The tabulated value for 95% confidence level of normal distribution is $z_{0.05/2} = z_{0.025} = 1.96$

Therefore, the margin of error is obtained below.

$$
\begin{aligned}
m &= z^* \frac{\sigma}{\sqrt{n}} \\
&= 1.96 \times \frac{6.5}{\sqrt{31}} \\
&= \boxed{2.28817\ U/l}
\end{aligned}
$$

95 % confidence interval for the mean for young women represented by the given sample is

$$
\begin{aligned}
&\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} \\
\text{Or } & 13.2 \pm 1.96 \times \frac{6.5}{\sqrt{31}} \\
\text{Or } & 13.2 \pm 2.28817 \\
\text{Or } & (10.91183, 15.48817) \\
\text{Or } & \boxed{(10.91183\ u/l, 15.48817\ u/l)}
\end{aligned}
$$

Therefore, 95% confidence interval for the mean for young women represent by the sample is between 10.91183 and 15.48817.

6.22 (1 point)

The given data is summarized as follows:

Number of apartments, $n = 10$

The sample mean monthly rent, $\bar{x} = \$980$

The sample standard deviation of rents, $\sigma = \$290$

We are supposed to find the 95% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent.

The critical value of *z* at 95% confidence level, $z^* = 1.96$

Assuming that monthly rents in Dallas have a Normal distribution, the 95% margin of error for $\mu$ is calculated using the following formula.

$$
\begin{aligned}
m &= z^* \frac{\sigma}{\sqrt{n}} \\
&= 1.96 \frac{290}{\sqrt{10}} \\
&= 179.7439
\end{aligned}
$$

Then, the 95% confidence interval for the mean monthly rent is calculated as follows:

$$\bar{x} \pm m = 980 \pm 179.7$$
$$= \boxed{(800.3, 1159.7)}$$

Hence, the 95% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent is ($800.3, $1159.7).

6.23 (1 point)

The 95% confidence interval (800.3, 1159.7) for monthly apartment rental in Dallas will probably not include 95% of all the unfurnished apartment rentals in the area. This is because the confidence interval is an estimate of the mean rent, not individual rents.

6.31 (1 point)

Use the margin of error for a level C interval, $m = z^* \frac{\sigma}{\sqrt{n}}$ to find the size needed.

Since the desired confidence is 95%, $z^* = 1.960$.

Now substitute in known information:

$$m = z^* \frac{\sigma}{\sqrt{n}}$$
$$1.5 = (1.960)\frac{6.5}{\sqrt{n}}$$
$$1.5 = \frac{12.74}{\sqrt{n}}$$

Solve for *n*:

$$\sqrt{n} = \frac{12.74}{1.5}$$
$$\sqrt{n} = 8.493$$
$$n = 72.131$$

Since a fraction of study cannot be taken, round up to the nearest whole number. Therefore,

$$\boxed{n = 73}$$

6.35 (2 points, one for each)

(a)

Let *X* be the number of intervals covers the true means.

Let the sample size be $n=5$ months.

Let the probability of interval cover the true means be $p=0.95$.

Here, *X* follows a binomial distribution with parameters $n = 5$ and $p = 0.95$. Then the probability mass function of a binomial distribution is,

$$P(X=x)=\binom{n}{x}(p)^x(1-p)^{n-x} \text{ ...... (1)}$$

Find the probability that all five intervals cover the true means.

Substitute 5 for *x* and 5 for *n* in equation (1).

$$\begin{aligned}P(X=5)&=\binom{5}{5}(0.95)^5(0.05)^{5-5}\\&=\frac{5!}{5!(5-5)!}(0.95)^5(0.05)^0\\&=0.7738\end{aligned}$$

Therefore, the probability that all five intervals cover the true means is 0.7738.

(b)

Find the probability that at least four intervals cover the true means.

$$\begin{aligned}P(X\ge 4)&=P(X=4)+P(X=5)\\&=\binom{5}{4}(0.95)^4(0.05)^{5-4}+\binom{5}{5}(0.95)^5(0.05)^{5-5}\\&=0.2036+0.7738\\&=0.9774\end{aligned}$$

Therefore, the probability that at least four intervals cover the true means is 0.9774.