2019Spring-ST305-HW4-Solution

(Total points: 36)

Ch 4.2: 21, 25, 26, 28, 31, 35, 36

4.21 **(1 point)**
Given that there are 5 different links. There will be 6 possible links in the web page. There will be links for all given 5 links and there will be another link to leave the page. Hence, the sample space for the outcome of a visitor to the Web page is {link1, link2, link3, link4, link5, leave}.

4.25 **(2 points: 1 for each)**
(a) The given probabilities have sum 0.97, so $P(\text{type AB}) = 0.03$.
(b) $P(\text{type O or B}) = 0.44 + 0.11 = 0.55$.

4.26 **(2 points: 1 for each)**
$P(\text{both are type O}) = (0.44)(0.52) = 0.2288$;
$P(\text{both are the same type}) = (0.42)(0.35) + (0.11)(0.10) + (0.03)(0.03) + (0.44)(0.52) = 0.3877$.

4.28 **(2 points: 1 for each)**
(a) $P(\text{French}) = 1 - P(\text{English}) - P(\text{Asian|Pacific}) - P(\text{Other}) = 1 - 0.59 - 0.07 - 0.11 = 0.23$.
(b) $P(\text{Canadian's mother tongue is not English}) = 1 - P(\text{English}) = 1 - 0.59 = 0.41$.

4.31 **(1 point)**
4.31. For example, the probability for A-positive blood is $(0.42)(0.84) = 0.3528$ and for A-negative $(0.42)(0.16) = 0.0672$.

Blood type	A+	A–	B+	B–	AB+	AB–	O+	O–
Probability	0.3528	0.0672	0.0924	0.0176	0.0252	0.0048	0.3696	0.0704

4.35 **(1 point)**
4.35. $P(\text{none are O-negative}) = (1 - 0.07)^{10} \doteq 0.4840$, so $P(\text{at least one is O-negative}) \doteq 1 - 0.4840 = 0.5160$.

4.36 **(1 point)**
If we assume that each site is independent of the others (and that they can be considered as a random sample from the collection of sites referenced in scientific journals), then

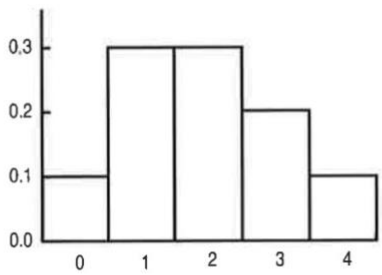
$$P(\text{all seven are still good}) = 0.87^7 = 0.3773.$$

Ch 4.3: 55, 56, 58, 62, 63, 64

4.55 **(3 points: 1 for each)**
4.55. **(a)** Histogram on the right. **(b)** "At least one nonword error" is the event "$X \geq 1$" (or "$X > 0$"). $P(X \geq 1) = 1 - P(X = 0) = 0.9$. **(c)** "$X \leq 2$" is "no more than two nonword errors," or "fewer than three nonword errors."

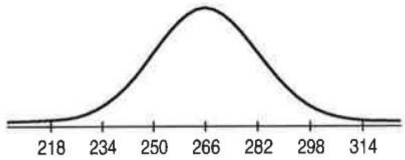
$$P(X \leq 2) = 0.7 = P(X = 0) + P(X = 1) + P(X = 2) = 0.1 + 0.3 + 0.3$$

$$P(X < 2) = 0.4 = P(X = 0) + P(X = 1) = 0.1 + 0.3$$



4.56 **(2 points: 1 for each)**

4.56. **(a)** Curve on the right. A good procedure is to draw the curve first, locate the points where the curvature changes, then mark the horizontal axis. Students may at first make mistakes like drawing a half-circle instead of the correct "bell-shaped" curve or being careless about locating the standard deviation. **(b)** About 0.81: $P(Y \leq 280) = P\left(\frac{Y-266}{16} \leq \frac{280-266}{16}\right) = P(Z \leq 0.875)$. Software gives 0.8092; Table A gives 0.8078 for 0.87 and 0.8106 for 0.88 (so the average is again 0.8092).



4.58 (1 point)

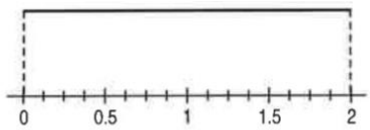
4.58. The possible values of Y are 1, 2, 3, . . . , 12, each with probability 1/12. Aside from drawing a diagram showing all the possible combinations, one can reason that the first (regular) die is equally likely to show any number from 1 through 6. Half of the time, the second roll shows 0, and the rest of the time it shows 6. Each possible outcome therefore has probability $\frac{1}{6} \cdot \frac{1}{2}$.

4.62 (5 points: 1 for each)

(a) $P(X \geq 0.30) = 0.7$.
(b) $P(X = 0.30) = 0$.
(c) $P(0.30 < X < 1.30) = P(0.30 < X < 1) = 0.7$.
(d) $P(0.20 \leq X \leq 0.25 \; or \; 0.7 \leq X \leq 0.9) = 0.05 + 0.2 = 0.25$.
(e) $P(\text{not}[0.4 \leq X \leq 0.7]) = 1 - P(0.4 \leq X \leq 0.7) = 1 - 0.3 = 0.7$.

4.63 (4 points: 1 for each)

4.63. **(a)** The height should be $\frac{1}{2}$ since the area under the curve must be 1. The density curve is at the right. **(b)** $P(Y \leq 1.6) = \frac{1.6}{2} = 0.8$. **(c)** $P(0.5 < Y < 1.7) = \frac{1.2}{2} = 0.6$. **(d)** $P(Y \geq 0.95) = \frac{1.05}{2} = 0.525$.



4.64 (3 points: 1 for each)

(a)

Calculate the area under the curve as follows:

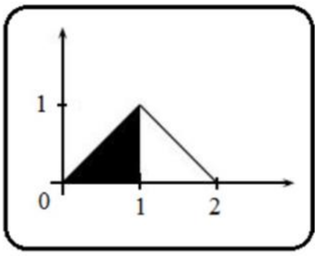
$$\begin{aligned} \text{Area} &= \frac{1}{2}bh \quad [b = \text{base and } h = \text{height}] \\ &= \frac{1}{2}(2)(1) \quad [\text{From the triangle}] \\ &= \frac{2}{2} \\ &= \boxed{1} \end{aligned}$$

(b)

Calculate $P(Y<1)$ as follows:

$$P(Y<1) = \frac{1}{2}bh$$
$$= \frac{1}{2}(1)(1) \quad \text{[From the below graph]}$$
$$= \boxed{\frac{1}{2}}$$

Required area is shown in below.

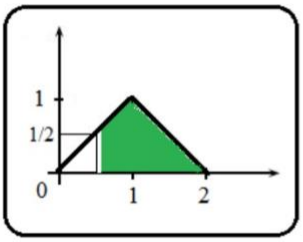


(c)

Calculate the probability that Y is greater than 0.6

$$P(Y>0.6) = 1 - P(Y \le 0.6)$$
$$= 1 - \frac{1}{2}(0.6)(0.6)$$
$$= 1 - 0.18 \quad \text{[From the below graph]}$$
$$= \boxed{0.82}$$

Required area is shown in below.



Ch 4.4: 75, 76, 78, 82, 93, 94

4.75 **(1 point)**

The average grade is $\mu = (0)(0.05) + (1)(0.04) + (2)(0.20) + (3)(0.40) + (4)(0.31) = 2.88$.

4.76 **(1 point)**

4.76. The means are

$$(0)(0.1) + (1)(0.3) + (2)(0.3) + (3)(0.2) + (4)(0.1) = 1.9 \text{ nonword errors and}$$
$$(0)(0.4) + (1)(0.3) + (2)(0.2) + (3)(0.1) = 1 \text{ word error}$$

4.78 **(1 point)**

4.78. In the solution to Exercise 4.75, we found the average grade was $\mu = 2.88$, so

$$\sigma^2 = (0 - 2.88)^2(0.05) + (1 - 2.88)^2(0.04)$$
$$+ (2 - 2.88)^2(0.2) + (3 - 2.88)^2(0.4) + (4 - 2.88)^2(0.31) = 1.1056,$$

and the standard deviation is $\sigma = \sqrt{1.1056} \doteq 1.0515$.

4.82 **(2 points: 1 for each)**

4.82. Let N and W be nonword and word error counts. In Exercise 4.76, we found $\mu_N = 1.9$ errors and $\mu_W = 1$ error. The variances of these distributions are $\sigma_N^2 = 1.29$ and $\sigma_W^2 = 1$, so the standard deviations are $\sigma_N \doteq 1.1358$ errors and $\sigma_W = 1$ error. The mean total error count is $\mu_N + \mu_W = 2.9$ errors for both cases. **(a)** If error counts are independent (so that $\rho = 0$), $\sigma_{N+W}^2 = \sigma_N^2 + \sigma_W^2 = 2.29$ and $\sigma_{N+W} \doteq 1.5133$ errors. (Note that we add the *variances*, not the standard deviations.) **(b)** With $\rho = 0.5$, $\sigma_{N+W}^2 = \sigma_N^2 + \sigma_W^2 + 2\rho\sigma_N\sigma_W \doteq 2.29 + 1.1358 = 3.4258$ and $\sigma_{N+W} \doteq 1.8509$ errors.

4.93 (2 points; one for each)

4.93. (a) Add up the given probabilities and subtract from 1; this gives P(man does not die in the next five years) $= 0.99749$. **(b)** The distribution of income (or loss) is given below. Multiplying each possible value by its probability gives the mean intake $\mu \doteq \$623.22$.

Age at death	21	22	23	24	25	Survives
Loss or income	−\$99,825	−\$99,650	−\$99,475	−\$99,300	−\$99,125	\$875
Probability	0.00039	0.00044	0.00051	0.00057	0.00060	0.99749

4.94 (1 point)

4.94. The mean μ of the company's "winnings" (premiums) and their "losses" (insurance claims) is positive. Even though the company will lose a large amount of money on a small number of policyholders who die, it will gain a small amount on the majority. The law of large numbers says that the average "winnings" minus "losses" should be close to μ, and overall the company will almost certainly show a profit.