$\bar{X} = 83.6$

$M = 86.5$

9	18
8	21
7	5
6	7
5	1

ST 305: Exam 2

By handing in this completed exam, I state that I have neither given nor received assistance from another person during the exam period. I have used no resources other than the exam itself and the basic mathematical functions of a calculator (ie, no notes, electronic communication, notes stored in calculator memory, etc.) Using your calculator for values from probability distributions like the normal or t is OK; however, if you are doing a calculation from a normal distribution show your work all the way to the point of calculating z-scores. I have not copied from another person's paper. I understand that the penalty if I am found guilty of any such cheating will include failure of the course and a report to the NCSU Office of Student Conduct. **I understand that I must show all work/calculations, even if they seem trivial, to get credit for my answers.**

Name: KEY

ID#: _____

$$\bar{x} = \frac{1}{n}\sum x_i$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$Z = \frac{X - \mu}{\sigma}$$

$$r = \frac{\sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)}{n-1}$$

$$b_1 = r\frac{s_y}{s_x}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$\text{residual} = y - \hat{y}$$

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A^C) = 1 - P(A)$$

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$\mu_X = \sum x_i p_i$$

$$\mu_{a+bX} = a + b\mu_X$$

$$\mu_{X+Y} = \mu_X + \mu_Y$$

$$\sigma_X^2 = \sum (x_i - \mu_X)^2 p_i$$

$$\sigma_{a+bX}^2 = b^2\sigma_X^2$$

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$$

$$\sigma_{X-Y}^2 = \sigma_X^2 + \sigma_Y^2$$

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y$$

$$\sigma_{X-Y}^2 = \sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y$$

$$\mu_X = np$$

$$\sigma_X = \sqrt{np(1-p)}$$

$$\hat{p} = X/n$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$P(X = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$$\mu_{\bar{X}} = \mu$$

$$\sigma_{\bar{X}} = \sigma/\sqrt{n}$$

Definitions. (5 points each) Clearly define each of the following terms.

1. Random variable:
a numerical result of a random phenomenon

2. Correlation:
measure of the strength & direction of a linear relationship between two variables

3. Simple Random Sample:
sample selected in such a way that every subset of n individuals from a pop'n has an equal chance of selection

Multiple Choice. (3 points each) Select the one best answer.

4. Nonresponse occurs when
A a. a portion of the individuals in a sample refuse to provide information
b. a portion of the population is not included in the sampling plan
c. a portion of the individuals in a sample give incorrect or misleading information

5. The collection of individuals from which a statistic is computed is
a. a census
B b. a sample
c. a population

6. If two events A and B are not disjoint, then
a. A and B are definitely independent
C b. A and B are definitely not independent
c. A and B may or may not be independent; it depends on the setting

7. Least squares regression should not be used if
a. there is a weak relationship between the explanatory and response variables
B b. the relationship between the response and explanatory variables is curved ← best answer
c. it is unclear which variable is explanatory and which is response ← I'm accepting this also

8. A scatterplot with correlation near 1 will lead to a least squares line with
C a. a positive, very steep slope
b. a positive, very shallow slope
c. a positive slope, but there is not enough information to tell how steep it will be

For the remaining problems, SHOW YOUR WORK. Numerical answers with no supporting work or explanation will receive zero credit, even if the calculations are trivial.

9. In each of the following experimental designs, one of the three key design principles is absent. Name the missing design element, and briefly explain how you could repair the design.

   a) A SRS of n=100 people is chosen for a study of the impact of daily meditation on blood pressure. The women in the sample are taught to use meditation daily, while the men are used as a control group and do not meditate. (5 points)

Randomization is missing. Randomly assign treatments without regard to sex – or carry out a blocked design instead

   b) To find out if the mean captive lifespan of Monarch butterflies is affected by the presence of lead in their drinking water, a SRS of n=75 Monarch butterflies are provided a diet with lead in their water. Their age at death is recorded. (5 points)

Control is missing. Provide some butterflies a diet with no lead

10. The following is the probability distribution for the number of days of rain in a calendar week in Topeka, Kansas:

Days	0	1	2	3	4	5	6	7
	0.5	0.2	0.1	0.1	0.04	0.03	0.02	0.01

a. What is the random variable in this example? (2 points)

# of days of rain in a week

b. What is the sample space for the random variable? (2 points)

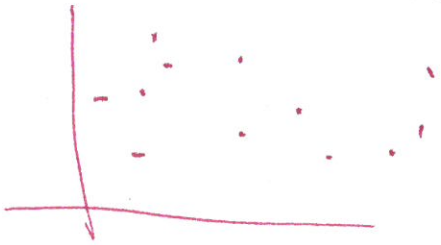
$S = \{0, 1, 2, \ldots, 7\}$

c. Is this a valid probability distribution? Justify your answer. (2 points)

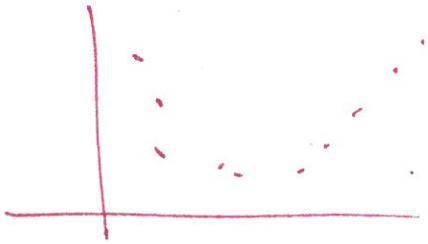
yes: ① all probabilities are between 0 & 1
② probabilities sum to 1

11. For each of the following, draw a scatterplot consistent with the description.

a. A weak, negative correlation. (3 points)



b. A strong, nonlinear relationship. (3 points)



c. Two variables with correlation coefficient approximately -0.9. (3 points)

12. An experiment is conducted to study the effect of dietary fat on cholesterol level. It is conducted as follows:

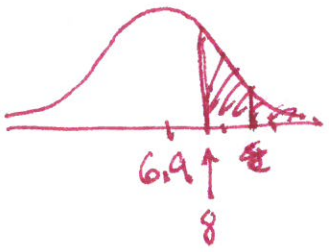
- A SRS of 50 men and 50 women are selected
- Of those 100 people, 25 random men and 25 random women eat a high fat diet for 6 months. The rest eat a normal diet.
- At the end of 6 months, the individuals' cholesterol levels are measured.

Identify the following elements of this experiment, if present (2 point each):

- Subjects: the 100 people in the study
- Factors: sex; diet
- Levels: M/F; normal/hi fat
- Explanatory variable: diet
- Response variable: cholesterol level
- Control group: individuals eating the normal diet
- Blocks: M/F (sex is a blocking factor)

13. The amount of sleep for American men follows a normal distribution with mean 6.9 hours and standard deviation 1.5 hours. I take a SRS of 5 men (assume that their sleep amounts are independent).

a. Find the probability that a randomly chosen man gets more than 8 hours of sleep. (3 points)



$$z = \frac{8 - 6.9}{1.5} = 0.733$$

$$P(z > .733) \approx \boxed{.2327}$$

b. Find the probability that none of the 5 men get more than 8 hours of sleep. (5 points)

P(none get 8 or more)

$= P\{1^{st}$ gets 8 or less AND 2^{nd} gets 8 or less AND $\cdots 5^{th} \cdots\}$

$= P\{1^{st}$ gets 8 or less$\} \times \cdots \times P\{5^{th}$ gets 8 or less$\}$

$= (1 - .2327) \times (1 - .2327) \times \cdots \times (1 - .2327)$

$= \boxed{.266}$

c. Find the probability that at least one of the 5 men gets more than 8 hours of sleep. (5 points)

P(at least 1 gets 8 or more)

$= 1 - P$(none get 8 or more) <Complement Rule>

$= 1 - .266 = \boxed{.734}$

14. A SRS of 143 black bears was taken, and for each bear the weight and the length of its head were recorded. The mean and standard deviation for the head lengths of the 143 bears was 13.42 inches and 1.92 inches, respectively. The mean and standard deviation for their weights was 192.16 pounds and 110.55 pounds. The correlation coefficient was 0.8333. A scatterplot suggested that the relationship between these two variables was linear. We want to use head length to predict a bear's weight.

a. Which variable was used as explanatory, and which was the response? Explain briefly how you can tell. (2 points)

Expl: head length Resp: weight

Head length is used to predict a bear's weight

b. Does there appear to be a strong linear relationship between Head Length and Weight? Justify your answer with numerical evidence from the information above. (3 points)

yes, the correlation coefficient is $r = 0.83$, indicating a strong positive linear relationship.

c. Find the least squares regression line. (Show your work on the back of this page.) (5 points)

$b_1 = r\frac{s_y}{s_x} = (.8333)\frac{110.55}{1.92} = 47.98$

$b_0 = \bar{y} - b_1\bar{x} = 192.16 - 47.98(13.42) = -451.73$

$\Rightarrow \hat{y} = -451.73 + 47.98x$

c. Predict the weight of a bear with a 14 inch head. (5 points)

$\hat{y} = -451.73 + 47.98(14) = 219.99$ lbs

d. What percentage of the total variation in the weight of these bears is explained by their head lengths? (3 points)

$r^2 = (.8333)^2 = .6944$

$\Rightarrow$ 69.44%

Table entry for z is the area under the standard normal curve to the left of z.

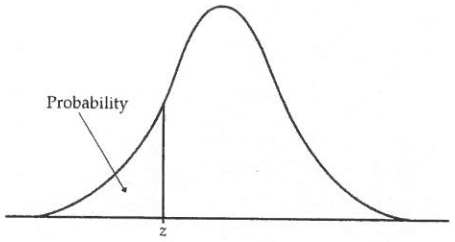


TABLE A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the area under the standard normal curve to the left of z.

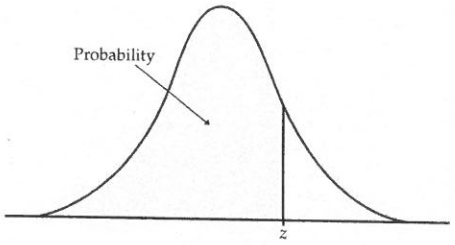


TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998